Solving linear equations and inequalities

Part A: Solve the following equations:

1) $3x - 8 = 13$
2) $4(n + 5) = 8$
3) $\frac{x}{3} - 4 = 5$
4) $2(7 - p) = 18$
5) $3n + 5 = 4n - 3$
6) $10 - 6a = 2a - 2$
7) $4(2 - x) = 5(2x + 3)$
8) $\frac{20}{n-1} = 4$
9) $\frac{2x-4}{2} + \frac{x-1}{6} = 6$
10) $\frac{2x+1}{3} + \frac{x-1}{6} = \frac{7}{2}$
11) $\frac{3}{x+6} = \frac{2}{x+5}$
12) $\frac{4}{x+2} = \frac{9}{3x+5}$

Part B: Solve the following inequalities:

1) $3x - 8 < 13$
2) $6 - \frac{x}{3} > -4$
3) $4(2 - x) \geq 5(2x + 3)$
4) $\frac{20}{n-1} < 4$
5) $2 < \frac{x}{3} + 1 < 5$
6) $0 < 2(7 - p) \leq 18$

Solutions:

Part A:

1) $3x - 8 = 13$ => $3x = 13 + 8 = 21$ => $x = 21 \div 3 = 7$

2) $4(n + 5) = 8$ [÷ 4 both sides] => $n + 5 = 2$ => $n = 2 - 5 = -3$

3) $\frac{x}{3} - 4 = 5$ => $\frac{x}{3} = 5 + 4 = 9$ => $x = 9 \times 3 = 27$

4) $2(7 - p) = 18$ [÷ 4 both sides] => $7 - p = 9$ => $7 = 9 + p$ => $p = 7 - 9 = -2$

5) $3n + 5 = 4n - 3$ [– 3n and + 3 both sides] => $5 + 3 = 4n - 3n$ => $8 = n$

6) $10 - 6a = 2a - 2$ [+ 6a and + 2 both sides] => $10 + 2 = 2a + 6a$ => $12 = 8a$ => $a = \frac{12}{8} = \frac{3}{2}$

7) $4(2 - x) = 5(2x + 3)$ => $8 - 4x = 10x + 15$ => $8 - 15 = 10x + 4x$ => $-7 = 14x$ => $x = -\frac{1}{2}$

8) $\frac{20}{n-1} = 4$ [x (n – 1) both sides] => $20 = 4(n - 1)$ => $5 = n - 1$ => $6 = n$

9) $\frac{2x-4}{2} + \frac{x-1}{6} = 6$ [x 6 all through] => $\frac{6(2x-4)}{2} + \frac{6(x-1)}{6} = 6 \times 6$ => $3(2x - 4) + x - 1 = 36$

=> $6x - 12 + x - 1 = 36$ => $7x = 36 + 12 + 1 = 49$ => $x = 7$

10) $\frac{2x+1}{3} + \frac{x-1}{6} = \frac{7}{2}$ [x 6 all through] => $\frac{6(2x+1)}{3} + \frac{6(x-1)}{6} = \frac{6\times7}{2}$ => $2(2x + 1) + x - 1 = 21$

=> $4x + 2 + x - 1 = 21$ => $5x = 21 + 1 - 2 = 20$ => $x = 4$

11) $\frac{3}{x+6} = \frac{2}{x+5}$ [multiply both sides by both denominators] => $3(x + 5) = 2(x + 6)$

=> $3x + 15 = 2x + 12$ => $3x - 2x = 12 - 15$ => $x = -3$

12) $\frac{4}{x+2} = \frac{9}{3x+5}$ => $4(3x + 5) = 9(x + 2)$ => $12x + 20 = 9x + 18$ => $3x = -2$ => $x = -\frac{2}{3}$

Part B:

1) $3x - 8 < 13$ [+ 8] => $3x < 21$ => $x < 7$

2) $6 - \frac{x}{3} > -4$ [– 6] => $-\frac{x}{3} > -10$ [x –3] => $x < 30$

3) $4(2 - x) \geq 5(2x + 3)$ => $8 - 4x \geq 10x + 15$

=> $8 - 15 \geq 10x + 4x$ => $-7 \geq 14x$ => $\frac{-7}{14} \geq x$ => $x \leq -\frac{1}{2}$

OR $-4x - 10x \geq 15 - 8$ => $-14x \geq 7$ => $x \leq \frac{-7}{14}$ => $x \leq -\frac{1}{2}$

4) $\frac{20}{n-1} < 4$ [x (n – 1) both sides] => $20 < 4(n - 1)$ => $5 < n - 1$ => $6 < n$ => $n > 6$

5) $2 < \frac{x}{3} + 1 < 5$ [–1] => $1 < \frac{x}{3} < 4$ [x3] => $3 < x < 12$

6) $0 < 2(7 - p) \leq 18$ [÷2] => $0 < 7 - p \leq 9$ [–7] => $-7 < -p < 2$ [x –1]

(need to flip BOTH inequalities) => $7 > p > -2$ or $-2 < p < 7$